Mail: banishagupta2099@gmail.com
Website: https://iibf.info

THEORY OF PROBABILITY

- Probability means chance/s or **possibility of happening of an event.**
- Probability gives **a numerical measure of this chance or possibility**.
- Suppose it says that there is a 60% chance that rain may occur in this weekend, 60% or 0.6 is called the probability of raining.

FACTORIAL: In mathematics, Factorial is equal to the product of all positive integers which are less than or equal to a given positive integer. 5! = 1 x 2 x 3 x 4 x 5 = 120

PERMUTATIONS AND COMBINATIONS

- A permutation is the arrangement of objects in which order is the priority.
- **The fundamental difference between permutation and combination is the order of objects,**
- In permutation, the order of objects is very important, i.e., the arrangement must be in the stipulated order of the number of objects, taken only some or all at a time.
- The combination is the arrangement of objects in which order is irrelevant. It is used when you must select r things from n.

- The notation for permutation is P (n, r) or ^nP_r, denoting the number of permutations of n things when r things are selected at a time.

If there are three things a, b and c, then permutations of three things taken two at a time is denoted by P (3,2) or $^3P_2$

It is given by (a, b), (a, c), (b, c), (b, a), (c, a), (c, b) = 6

P (n, r) = $^nP_r = \frac{n!}{(n-r)!}$

Q. In how many ways 2 balls can be selected from 3 balls?

Solution: $^3C_2$ ways

Let's assume 3 balls be: a, b and c. Possible ways: (a, b), (b, c), (c, a)

$$= \frac{n!}{r!\,(n-r)!} = \frac{3!}{2!\,(3-2)!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} = 3$$

RANDOM EXPERIMENT OR TRIAL: An operation or experiment conducted under identical conditions, and which has a number of possible outcomes is called Random Experiment or Trial.

Example: 1. Tossing a coin 2. Throwing a dice 3. Selecting a card form a pack of cards.

SAMPLE SPACE (S) AND SAMPLE POINTS n(S): The set of all possible outcomes of a random experiment is called sample space.

The elements of the sample space are called sample points. Sample space is denoted by S.

Example: In an experiment of throwing a coin, S- [H, T], n(S)- 2

EVENT: Any subset of the sample space S is called an event. If S is a sample space and A is a subset of S (i.e., A ⊂ S), then A is called an event.

Example: In an experiment of throwing dice where S = {1, 2, 3, 4, 5, 6), the event of getting odd numbers is A (1,3,5)

TYPES OF EVENTS

CERTAIN EVENT: If sample points in an event are same as sample points in sample space of that random experiment, then the event is called a certain event. Example: Getting any number between1 to 6 on a dice is a certain event.

IMPOSSIBLE EVENTS: An event which never occurs, or which has no favourable outcomes is called an impossible event.

- The event corresponding to the set o (null set) is called an impossible event.
- Example: Getting a number 7 on a dice is an impossible event.

MUTUALLY EXCLUSIVE EVENTS: if the happening of any of them restricts the happening of the others

- if no two or more of them can happen together or simultaneously in the same trial.

Example: In tossing a coin event head and tail are mutually exclusive. **EQUALLY LIKELY EVENTS**: if they have equal choice to occur. Example: In throwing a dice all the six faces are equally likely to occur.

EXHAUSTIVE EVENTS: If the sample points of the events taken together constitute the sample space of the random experiment, the events are called exhaustive events.

Example: Random Experiment: Throwing a dice S = (1,2,3,4,5,6)
A Event of odd numbers= (1, 3, 5)
B= Event of even numbers = {2, 4, 6)
$A \cup B = (1, 2, 3, 4, 5, 6)$ = S, Here A and B are called exhaustive events.
COMPLEMENTARY EVENT: Two events A and B are called complementary events if A and B exhaustive as well as mutually exclusive events.

MATHEMATICAL DEFINITION OF PROBABILITY: If the sample space S of a random experiment consists of n equally likely, exhaustive and mutually exclusive sample points and m of them are favourable to an event A, then the probability of event A is given by
P(A)= m/n = number of sample point in A /number of sample point in S = n(A)/n(S)

Q: 2 unbiased coins are tossed simultaneously. What is the probability of getting at least 1 head.

S→{(H, H), (H, T), (T, T), (T, H}, n(S) = 4
P(A) = $\frac{n(A)}{n(s)} = \frac{3}{4}$

ADDITION THEOREM: Let A and B are two events (subsets of sample space S) and are not disjoint, then the probability of the occurrence of A or B or A and B both, in other words probability of occurrence of at least one of them is given by,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the events A and B are mutually exclusive, then

$P(A \cup B) = P(A) + P(B)$

For three non-mutually exclusive events A, B, C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

CONDITIONAL PROBABILITY: The **conditional probability of an event A** is the probability that the event will occur given the knowledge that an event B has already occurred.

Probability of the event A given the event B has already occurred and denote it by P (B/A).

If the events A and B are such that the occurrence of A doesn't depend upon occurrence of event B, (A and B are independent event), the conditional probability of event A given event B is simply the probability of event A, that is P (A).

MULTIPLICATION THEOREM

If A and B are two events of a sample space S associated with an experiment, then the probability of simultaneous occurrence of events A and B is given by

$$P(A \cap B) = P(A)\, P(B/A) = P(B)\, P(A/B)$$

INDEPENDENT EVENTS: Two events A and B are independent of each other if the occurrence or non-occurrence of one does not affect the occurrence of the other.

$$P(A \cap B) = P(A)\,P(B)$$

RANDOM VARIABLE

A random variable is a function that associates a real number with each element in the sample space.

Discrete Random Variable:

If a random variable takes a finite number or countable infinite number of possibilities, it is called a discrete random variable.

Example

1. Age in years
2. Number of arrivals in a clinic
3. Number of accidents

Continuous Random Variable: If a random variable takes infinite number of possibilities, it is called a continuous random variable.

Example

1. Percentage of marks
2. Weight

PROBABILITY DISTRIBUTION OF RANDOM VARIABLE

A probability distribution is a statistical function that describes **all the possible values of a random variable X** and **its corresponding probabilities that X can take within a given range**. This range will be bounded between the minimum and maximum possible values.

Example: Number of customers arrive in a bank at a particular time is a discrete random variable. The values of the random variable along with its corresponding probabilities is a discrete probability distribution and is called **Probability Mass Function (PMF).**

f(x) is a probability mass function of X if for each possible outcome of x,

$f(x) \geq 0$

1. $\Sigma f(x) = 1$

$P(X = x) = f(x)$

Example: Expected Returns of a particular stock is a **continuous random variable.** The range of values of the random variable with possible values (probabilities) is a continuous probability distribution and is called **Probability Density Function (PDF).**

Let f(x) represents PDF of X and f(x) dx represents the area bounded by the curve f(x), x axis and the ordinates at the points x - dx/2 and x + dx/2. Then,

$f(x) \geq 0$,

$f(x)dx = 1$

Cumulative Distribution Function of Random Variable X

Let X be a random variable with PMF or PDF f(x). $F(x) = P(X \leq x)$ is called Cumulative Distribution Function or Distribution function of X.

Properties

1. $0 \le F(x) \le 1$

2. $F(-\infty)=0$ and $F(+\infty) = +1$

3. F(x) is non decreasing function and continuous from right.

EXPECTATION AND STANDARD DEVIATION OF RANDOM VARIABLE

The **mean (the expected value) E(X) of the random variable (sometimes denoted as μ**) is the value that is **expected to occur per repetition,** if an experiment is repeated a large number of times.

Expectation is the average or mean of the distribution.

For the discrete random variable, E (X) is defined as-

$E(X) = \sum x f(x)$, where f(x) is the probability mass function of X.

The **variance V(X) of the random variable** X measures scatteredness of data (sometimes denoted as $S.D^2$) is defined by the formula.

$$V(X) = E(X^2) - [E(X)]^2$$

The variance of the discrete random variable is given by

$V(X)= \sum (x-E(X))^2 .f(x)$

Standard Deviation = $\sqrt{Variance}$

Properties of Expectation and Variance

$E (a X + b) = a E(X) + b$

$V (aX+b) = a^2V(x)$

BINOMIAL DISTRIBUTION

Consider a random experiment consisting of **n repeated independent trials with p the probability of success at each individual trial**. Let the random variable X represent the number of successes in the n repeated trials. Then X follows a Binomial distribution.

$f(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$; x = 0, 1, 2 ..., n

p + q = 1

P(x) is PMF [Probability function of mass]

Q: Find the probability of getting 3 heads from 5 no. of trials?

POISSON DISTRIBUTION

If the value of n is very large (n=∞) and the value of P is too small (p=0) and np is a finite number.

$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x = 0,1,2,3 \ldots .\infty$, [e=2.7183]

Q: Manufacturer who produces medicine bottles, find that 0.1% of the bottles are defective the bottles are packed in a box containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottle using poison distribution find how many boxes will contain no defective? $e^{-0.5}$ = 0.6065

n = 500 bottles

Probability of defective bottles P = 0.1%

P = 0.1/100 = 0.001

Mean = m = np = 500 X 0.001 = 0.5

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.5}0.5^{0}}{0!} = 0.6065$$

$$Mean = \ variance = \lambda$$

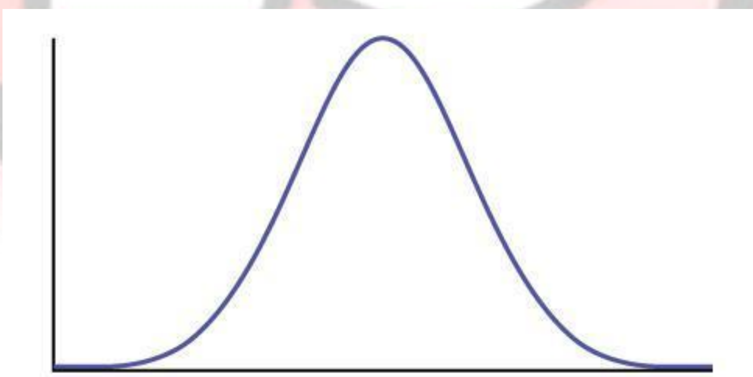
$$Measure\ of\ Skewness\ \beta_1 = \frac{1}{\lambda}$$

$$Measure\ of\ Kurtosis\ \beta_2 = 3 + \frac{1}{\lambda}$$

Mode: if λ is not an integer mode is integral part lying between $\lambda - 1\ to\ \lambda$. If λ is integer it will be bimodal and modes be $\lambda - 1\ and\ \lambda$.

NORMAL DISTRIBUTION (GAUSSIAN)

It is **a probability distribution that is symmetric about the mean**



In that case:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

μ = x̄

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	PROPERTY	EQUATION
Properties of Sampling Distribution of Mean, when population is normally distributed.	• **Sampling distribution has mean equal to population mean.** • **Sampling distribution has standard deviation equal to population standard deviation divided by square root of sample size.**	$\mu_{\bar{x}} = \bar{x}$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

PROPERTIES

a. If μ = 0 and $S.D^2 = 1$, then the Normal variable is called Standard Normal Variable.
b. The graph of Normal Distribution is bell shaped and symmetric.
c. The total area under normal curve is 1.
d. Quartile deviation is 0.6745
e. Mean deviation is 0.7979 s
f. Mean = Median = Mode = μ

Normal population of 1000 employees has mean income Rs. 800 per day and variance 400, Find no. of employees where income between

(i) P (750 < x < 820)
(ii) P (x > 700)
(iii) P (x > 760)

$$z = \frac{\bar{x} - \mu}{\sigma}$$

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CASE 1: P (750-800/20 < Z < 820-800/20)
= P(-2.5<Z<1) = 0.3413 + 0.4938 = 0.8351
No. of employees where income between 750 and 820 is 1000 * 0.8351 = 835

CASE 2: P (Z > 700-800/20)
= P(Z> -5) = 0.5 + 0.5 = 1
No. of employees where income is more than 700 is 1000 * 1 = 1000

CASE 3: P (Z > 760-800/20)
= P(Z> -2) = 0.5 + 0.4772 = 0.9772
No. of employees where income is more than 760 is 1000 * 0.9772= 977

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.01736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3565	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.03997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441

1.6	0.4452	0.4463	0.4457	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4938	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4915
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

CREDIT RISK

When lenders offer mortgages, credit cards, any type of loan to different customers, there could be risk that the customer or borrower might not repay the loan. This is called Credit Risk.

Thus, Credit Risk is the possibility or chance or probability of a loss occurring due to a borrower's failure to repay a loan to the lender or to satisfy contractual obligations.

There are three types of credit risks.

CREDIT DEFAULT RISK

Credit default risk is the type of loss that is incurred by the lender either when the borrower is unable to repay the amount in full or when 90 days pass the due date of the loan repayment. This type of credit risk is generally observed in financial transactions that are based on credit like loans, securities, bonds or derivatives.

CONCENTRATION RISK

Concentration risk is the type of risk that arises out of significant exposure **to any individual or group** because any adverse occurrence will have the potential to inflict large losses on the core operations of a bank. The concentration risk is usually associated with significant exposure to a single company or industry or individual.

COUNTRY RISK

The **risk of a government or central bank** being **unwilling or unable to meet its contractual obligations** is called Country or Sovereign Risk.

The expected loss is based on the value of the loan (i.e., the exposure at default, EAD) multiplied by the probability, that the borrower will default (i.e., probability of default, PD).
EL = PD X EAD X (1-PD)

Q: Let a credit of Rs. 2,000,000 was extended to a company one year ago. Determine the expected loss for the exposure if the company defaults completely, where the loss given default is 50%.

- Exposure at default, EAD = Rs. 2,000,000
- Probability of default, PD = 100% (as the company is assumed to be in default completely)
- Loss given default, LGD = 50%

Expected Loss
= 100% X Rs. 2,000,000 X (1 - 50%)
Expected Loss = Rs. 1,000,000

VALUE AT RISK (VaR)

- The concept of value at risk is associated with portfolio of an individual or an organisation.
- A portfolio is a collection of different kinds of assets owned by an individual or organisation to fulfil their financial objectives.
- One can include fixed deposit or any investment where he or she can earn a fixed interest, equity shares, mutual funds, debt funds, gold, property, derivatives, and more in his portfolio.

OPTIONS

One of the kinds of derivative is the options contract.
There are two types of options: the call option and the put option.
A **CALL OPTION** is a contract that gives the buyer the right, but not the obligation, to buy a particular asset at a specified price on a specific date.
PUT OPTION: In this type of contract, you can sell assets at an agreed price in the future, but not the obligation.

STRIKE PRICE/ EXERCISE PRICE

This is the price at which a call holder can buy stock and a put holder can sell it.

TIME TO MATURITY (t)

The **value of an option on the date of maturity** is just the **difference between the strike price and stock price.** This is the intrinsic value of the option.

On any day prior to the date of maturity, the option carries time value, in addition to the intrinsic value. As time to maturity decreases, time value also deceases.