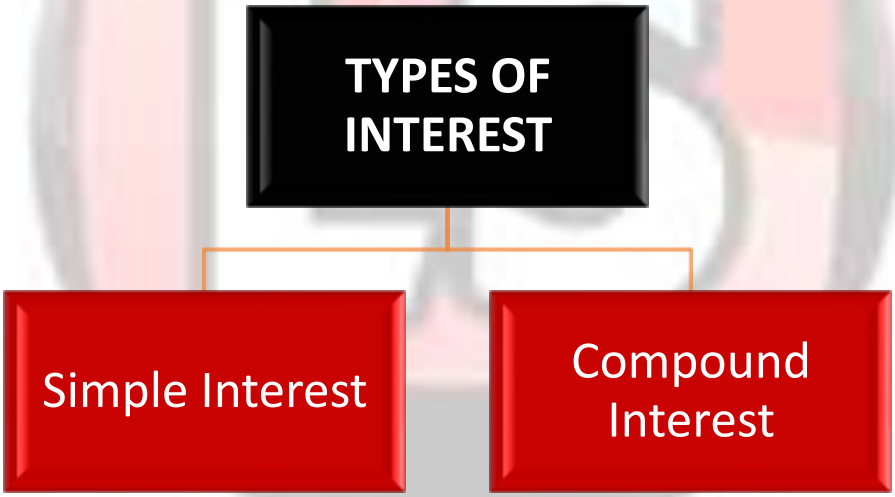
CALCULATION OF INTEREST AND ANNUITIES

INTEREST

- When money is lent, the **borrower usually pays a fee** to the lender. This fee is called 'interest'.
- Interest refers to the **cost of borrowing money or the return on investment** for lending money.



SIMPLE INTEREST

- Simple interest is calculated only on the **principal amount over a fixed period of time** and **charged only once in entire loan Period**.
- It **does not consider any additional interest** earned or accrued on previous interest payments.

Simple Interest = Principal x Rate x Time

- **Interest'** is the total amount of interest paid.
- **Principal'** is the amount lent or borrowed
- **Rate'** is the percentage of the principal chargeable as interest for one year.
- **Time'** is the time in years of the loan or deposit.

Q:1 A person purchases a car by obtaining a loan on simple interest. The car costs Rs 1,50,000 and the interest rate on the loan is 10 percent per annum (simple). If the loan is to be paid back after two years, calculate simple interest.

Solution

Simple Interest = Principal x Rate x Time

COMPOUND INTEREST

- Compound interest is an interest calculated **on the principal and the existing interest together** over a given time period.
- The Interest accumulated on a principal over a period of time is **also added to the principal and becomes the new principal** amount for the next time period.

$$.A = p(1 + r/n)^{n_t}$$

P = Principal (Initial amount you borrowed or deposited)

r= Annual rate of interest (per cent)

n = Number of compounding period for the year

A = Amount of money accumulated (Interest + principal)

COMPOUNDING PERIOD

- It refers to the **frequency** at which the i**nterest on an investment or loan is calculated and added to the principal amoun**t.
- Common compounding periods include **annually, semi-annually, quarterly, monthly, weekly, and daily**.

Interest is paid more frequently.

Annually = P (1 + r) = Annual compounding

Quarterly = $p\left(1+\frac{r}{4}\right)^{4}$ = Quarterly compounding

Monthly = $p\left(1+\frac{r}{12}\right)^{12}$ = Monthly compounding

Q:1 X invested Rs.100000 in a bank FDR at 6% p.a. for 2 year. If interest is compounded on half-yearly basis, the amount payable shall be?

Solution

$$A = p(1 + r/n)^{n_t}$$

The Rule of 72

- You can use the rule of 72 to **estimate the number of years** it takes for **your money to double** with yearly compounding.
- Simply **divide the number 72 by the annual interest rat**e (expressed as a percentage) you are paying on your debt or earning on your investment.

Ex: You borrowed Rs 1,0000 at 6 percent interest. Then, 72 divided by 6 is 12. That makes 12 the approximate number of years it would take for your debt to double to Rs 2,0000, if you did not make any payment.

The rule of 72 states that you can determine the approximate number of years it takes for your money to double by dividing 72 by the percentage rate. In this case, the percentage rate is 6 percent.

72 / 6 = 12

Explanation

Therefore, 12 is the approximate number of years it would take for your debt to double to Rs 2,000 if you did not make any payments.

FIXED AND FLOATING INTEREST RATES

Fixed Interest Rate

- It is an interest rate that **remains constant throughout the duration** of a loan or an investment.
- It **does not change over time**, regardless of **market fluctuation**s or changes in the economy.

Floating Interest Rate

- In the floating rate or variable rate, the rate of **interest changes, depending upon the market conditions.**
- floating rate is tied to an underlying **benchmark or reference rate**, such as the prime rate, LIBOR, or a government bond yield.

FRONT-END AND BACK-END INTEREST RATES

FRONT-END INTEREST

It refers to a scenario where the **interest is subtracted from the principal amount of a loan,** and the remaining net amount is disbursed to the borrower.

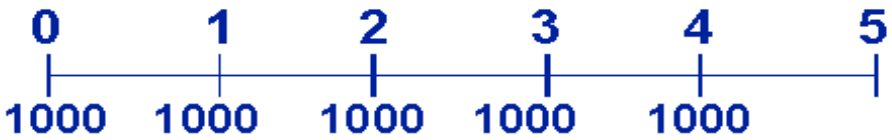
BACK-END INTEREST RATE

It refers to a situation **where the full loan amount is disbursed** to the borrower upfront, and the **interest is charged later on a periodic basis**, such as monthly, quarterly, or as agreed upon.

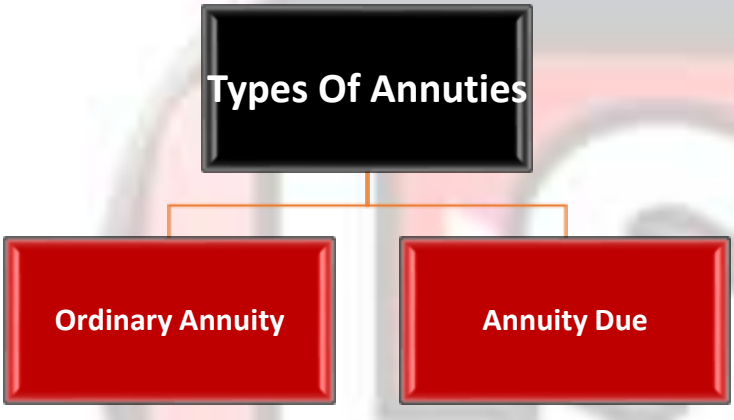
ANNUITIES

An **annuity** is a financial product that provides a **series of regular payments over a specified period of time**, typically in exchange for a lump sum payment or series of payments

Example



If a landlord collects monthly rental payments from tenants, it can be seen as an annuity. The landlord receives a fixed amount of income over a specified period, typically on a monthly basis.



Ordinary Annuity

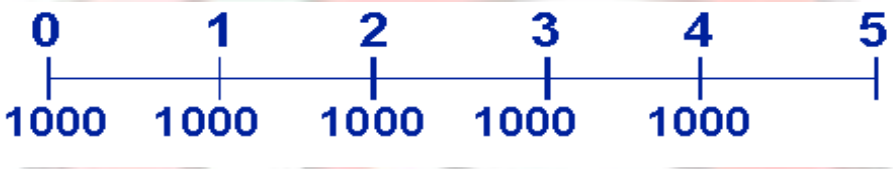
It refers to a series of **equal and regular cash flows or payments that occur at the end of each period** such as at the end of a month, year, or any other defined time interval.

Example

If you invest in a stock or a mutual fund that pays annual dividends, you will receive equal dividend payments at the end of each year then this is called ordinary Annuity.

Annuity Due

It refers to a series of **equal and regular cash flows** or payments where each payment is made at the **beginning of each period** such as at the beginning of a month, year.



Example

An example of an annuity due is a monthly rent payment. If you are a tenant, you would make equal monthly rental payments at the beginning of each month. Each payment represents the rent for that particular month.

FUTURE VALUE AND PRESENT VALUE OF ANNUITY

Future Value of an Annuity

It represents the **total value of a series of cash flows at a specified future date**. It shows accumulated value of a stream of payments in the future.

Present Value of an Annuity

It represents the **current value of a series of future cash flows**, discounted back to the present. It shows **current worth of a stream of future payments**

CALCULATING THE FUTURE VALUE OF AN ORDINARY ANNUITY

FV (ordinary annuity) = $C\left[\frac{(1+i)^n-1}{i}\right]$

C= Cash flow per period

I = Interest rate

n = number of payments

A opened a recurring account with a bank to deposit Rs.16000 by the end of each year at 10% interest rate. How much he would get at the end of 3rd year?

Solution

FV (ordinary annuity) = $C\left[\frac{(1+i)^n-1}{i}\right], = 16000\left[\frac{(1+0.10)^3-1}{0.10}\right]$

If a person is depositing Rs 10000 per year for 18 years in the end of each year at 10% per annum how much amount will be available with him at the end of 18 years

Solution

Future Value = $C\left[\frac{(1+i)^n-1}{i}\right], =$

CALCULATING THE PRESENT VALUE OF AN ORDINARY ANNUITY

$$PV\ of\ Ordinary\ Annuity = C\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right]$$

- C = Cash flow per period
- r = discount rate
- n = number of payments

Z received 20000 at the end of every six months for 5 years and interest rate 10 % interest per annum and compounded semi-annually. calculate present value of funds.

Solution

$$Present\ value = C\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right],$$

CALCULATING THE FUTURE VALUE OF AN ANNUITY DUE

$$FV\ of\ Annuity\ Due = C\left[\frac{(1+i)^n - 1}{i}\right] \times (1+i)$$

- C= Cash flow per period
- i = interest rate
- n = number of payments

Q:1 **If a person is depositing Rs 1000 per year for 18 years in the beginning of each year at 5% per annum how much amount will be available with him at the end of 18 years**

$$Future\ value = C\left[\frac{(1+i)^n - 1}{i}\right] \times (1+i)$$

CALCULATING THE PRESENT VALUE OF AN ANNUITY DUE

$$PV\ of\ Annuity\ Due = C\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right](1+r)$$

- C = Cash flow per period
- r = discount rate
- n = number of payments

Z received 20000 at the beginning of every six months for 5 years and interest rate 10 % interest per annum and compounded semi-annually. calculate present value of funds.

Solution

$$Present\ value = C\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right](1+r)$$

REPAYMENT OF A DEBT

It refers to the act of r**eturning borrowed money or fulfilling a financial obligation** to a lender or creditor.

METHODS

Equal monthly/ quarterly installment of principal plus the interest applied during the period.

Ex: Your friend has borrowed Rs 1,000 from you and wants to repay you on a monthly basis rather than the whole amount all at once at the end of the year. The important point here is that he will owe you less in principal each month. The applicable rate of interest, 12% p.a. means 1 % per month.

Balance	Principal Payment	Interest Payment	Total Payment
1000	200	10	210
800	200	8	208
600	200	6	206
400	200	4	204
200	200	2	202
Total	1000		1030

Equated monthly/ quarterly installment covering both the principal and the interest

$$EMI = \frac{P \times r \times (1+r)^n}{(1+r)^n - 1}$$

where P = principal amount, r = rate of interest per instalment period, n = no. of instalments in the tenure.

Q:1 For a loan of Rs 1,00,000, at an interest rate of 10% p.a., to be repaid in 2 years, the EMI is _____.

Solution: -

First, we calculate No. of installments and monthly interest rate.

No. of payments = 2x12=24

Semi-annually interest rate=10%/12=0.83%

$$EMI = \frac{P \times r \times (1+r)^n}{(1+r)^n - 1}$$

Bullet/ balloon repayment under which the entire loan amount is repaid at the end of the period (creation of sinking fund)

If the entire loan amount is repaid at the end of the period with accumulated interest, the amount can be easily calculated by applying the formula for compound interest.

If the interest is paid periodically, as and when applied, the last installment will be equal to the loan amount and the interest for the last period.

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